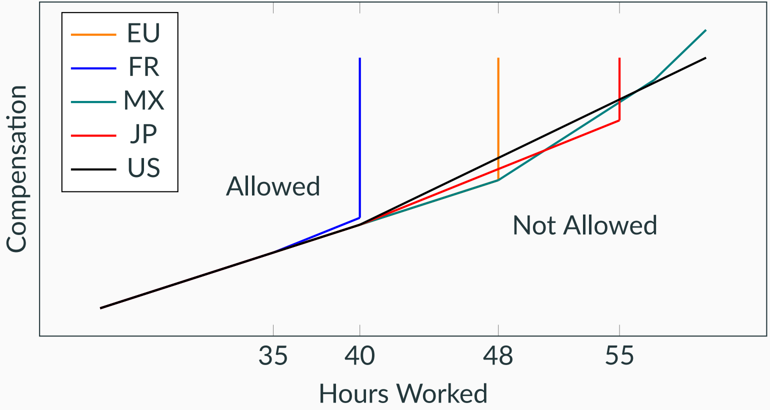
Regulation of Wages and Hours

Matthew W. Thomas

Northwestern University

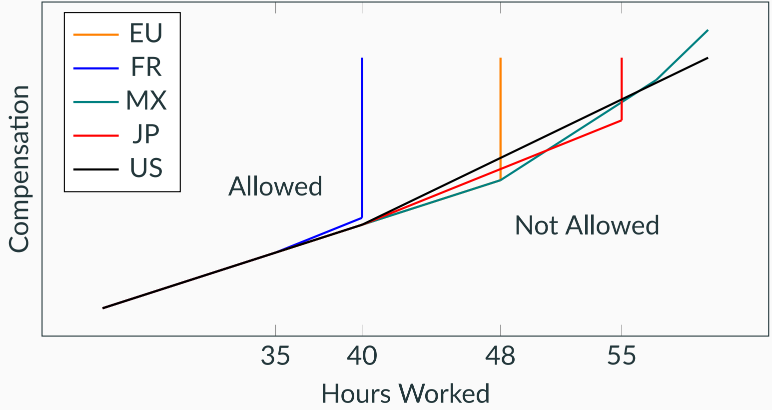
Introduction

Overtime and hours caps



Regulations intended to reduce workers' hours

Overtime and hours caps: understand and refine



Such regulations are common and heterogeneous: **Why? What is optimal?**

Regulating wages and hours

To study, need model of hours bargaining and regulation

- *Pareto efficient* joint bargaining of hours and wages
- *Redistributive* regulation that restricts bargaining space

Overtime, hours caps, and minimum wage are examples of such regulations

Preview and example

Jardim et al. (2022) study effects of 2014 minimum wage increase in Seattle[1]

- Find significant reductions in hours for individual workers
- Hours reductions are considered *bad*
- *In some cases, workers may want their hours to be reduced*

[1] Pandit (2023) finds similar effects for other minimum wage increases

Outline

1. **Complete information: minimum wage optimal**
    - Efficient joint contracting $\implies$ labor often not on supply or demand curve
    - Labor hours may exceed total surplus maximizing level
    - Alters intuition about relationship between labor hours and total surplus

Outline

1. Complete information: minimum wage optimal
2. **Robust setting: optimal minimum wage, overtime, and hours cap**
   - No exogenous bounds are enforced on parameters
   - Instead, *endogenous* bounds from individual rationality of **preexisting market state**

Related literature

Optimal minimum wage regulation without contracted hours

- Berger et al. (2022), Flinn (2006), and Stigler (1946)

Contracted hours without regulation

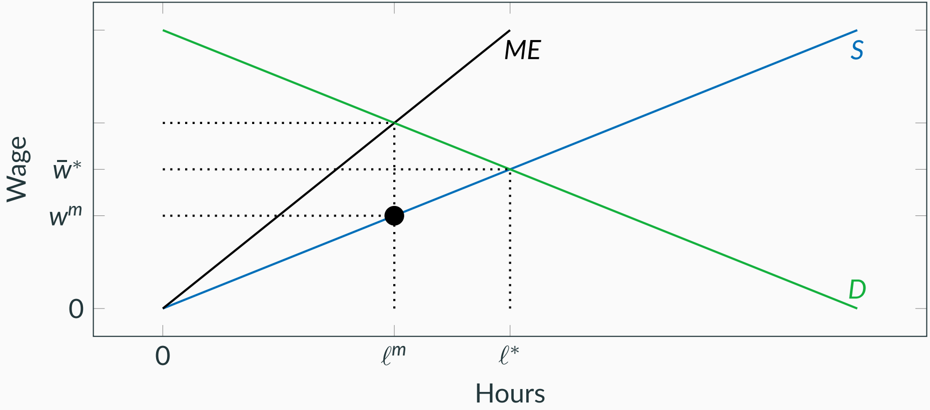
- Altonji and Paxson (1988), Feather and Shaw (2000), and Manning (2013)

Empirical: effects of hours-based regulation

- Crépon and Kramarz (2002), Hamermesh and Trejo (2000), and Trejo (1991)

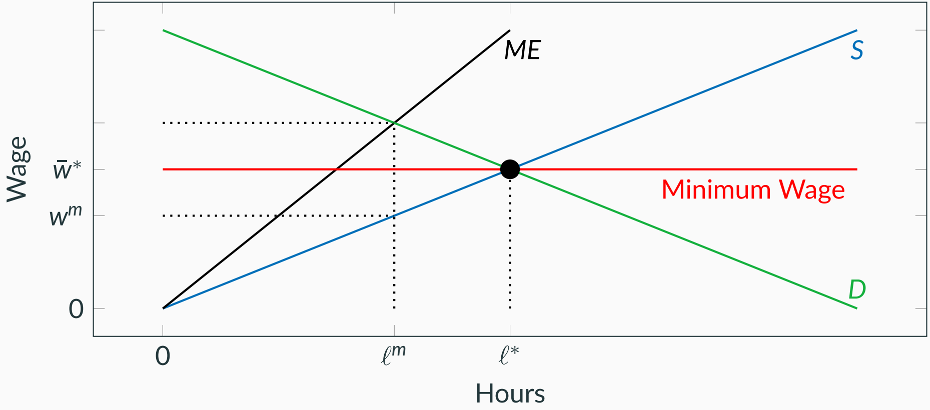
Flexible-hours model

Canonical flexible-hours model of monopsony



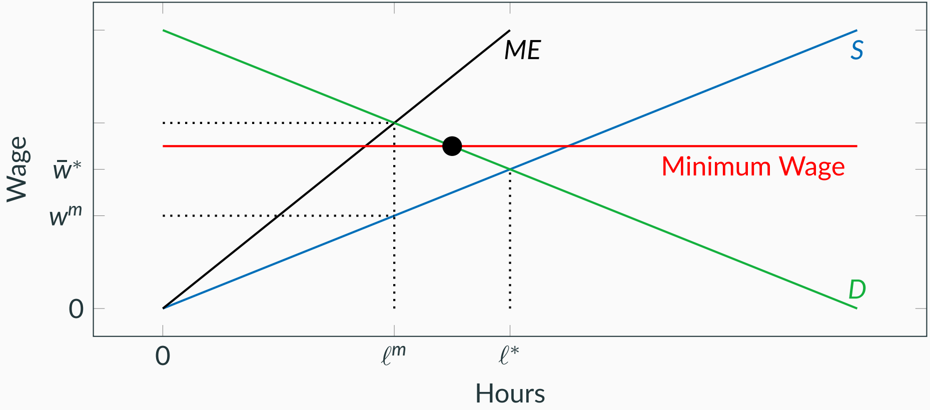
Worker chooses hours at posted wage: *hours not contractible*

Canonical flexible-hours model of monopsony



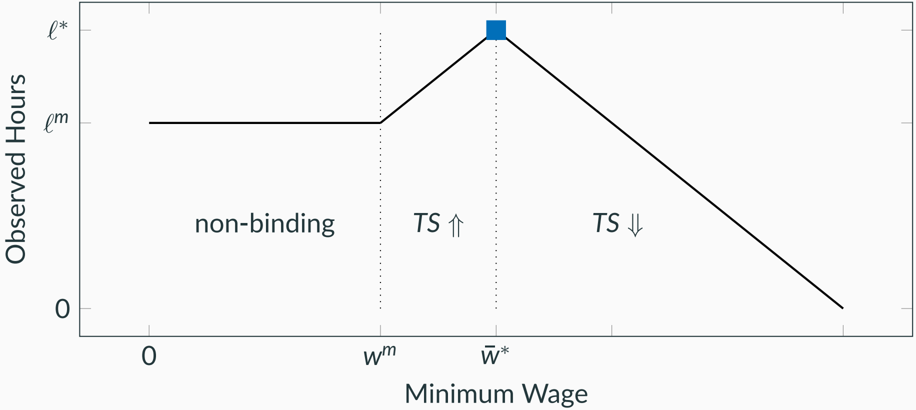
Minimum wage can increase labor to TS maximizing level

Canonical flexible-hours model of monopsony



Labor hours decrease in minimum wage after TS maximizing point

Effect of minimum wage on labor and total surplus



Increasing/maximizing hours and increasing/maximizing total surplus are equivalent

Ultimatum bargaining model

Ultimatum framework

- One firm contracts with one worker (extend later)
- Contract (ℓ, τ): worker works ℓ hours for total compensation τ
- Firm makes "take it or leave it" offer[2] under complete information
- Firm profits

$$\pi(\ell, \tau) = f(\ell) - \tau,$$

worker payoff

$$u(\ell, \tau) = \tau - c(\ell).$$

[2] In paper, allow for more general bargaining.

Ultimatum framework

- Firm makes "take it or leave it" offer[2] under complete information
- Firm profits

$$\pi(\ell, \tau) = f(\ell) - \tau,$$

worker payoff

$$u(\ell, \tau) = \tau - c(\ell).$$

Assume:

$f, -c, -c'(x)x$ strictly concave, differentiable, $f'(0) > c'(0) > 0 > \lim_{x \to \infty} f'(x) - c'(x)$

[2] In paper, allow for more general bargaining.

Wage and overwork

Definition (Wage)

Worker's wage is compensation per hour: $w \equiv \tau/\ell$

Definition (Overwork)

Worker is overworked if she would prefer to work fewer hours for the same wage:

$$\text{wage} \;<\; \text{marginal cost}$$

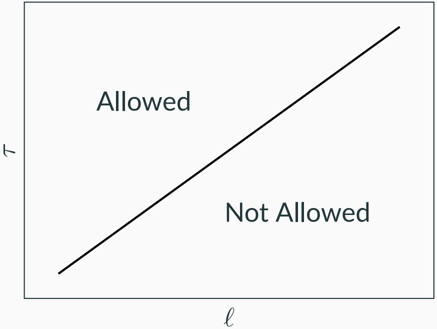
Regulation/delegation

Definition (Regulation)

A convex function of hours, $\phi : \mathbb{R}_+ \to [0, \infty]$, s.t. contracts in $\{(\ell, \tau) : \tau < \phi(\ell)\}$ are forbidden.

Definition (Minimum wage)

The slope of a linear policy. That is, $\bar{w}$ is the minimum wage if $\phi(x) \equiv \bar{w}x$.



Objective of regulation

Regulator's objective:

Maximize total surplus and break ties in favor of worker[3]

[3] More aggressive redistribution considered later

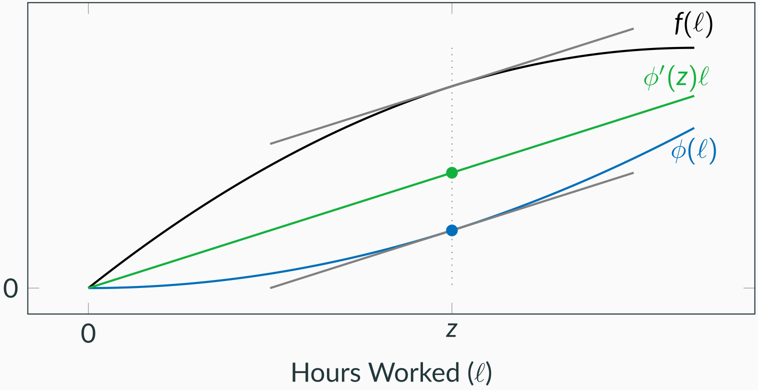
Results

Overwork

Ultimatum game without regulation:

- Firm extracts all surplus
- Total surplus is maximized
- Wage is worker's average cost
- Worker is overworked (average cost $<$ marginal cost)

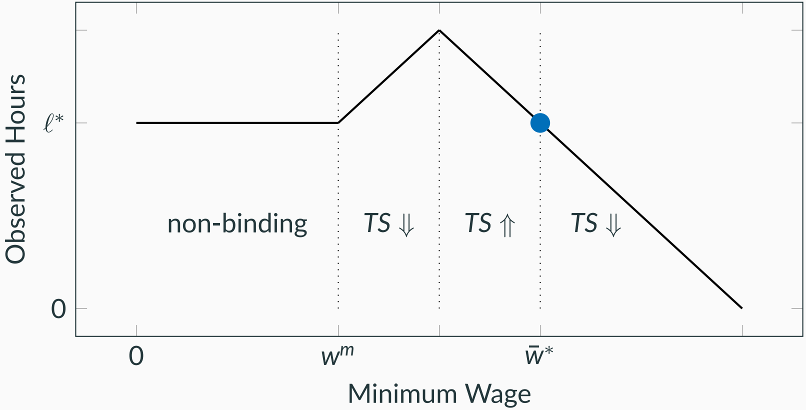
Minimum wage maximizes worker utility



Minimum wage is first best

If ϕ results in z hours, minimum wage $\phi'(z)$ results in z hours with more compensation

Effect of minimum wage on hours and total surplus in ultimatum model



Increasing/maximizing hours and increasing/maximizing total surplus **not** equivalent

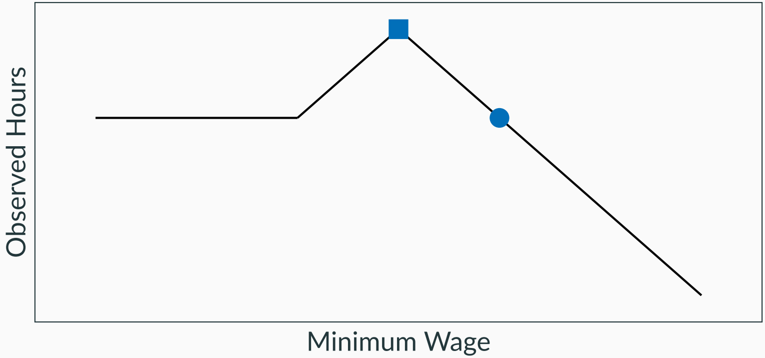
Models are "indistinguishable"

Remark

Flexible-hours model generates same labor curve as ultimatum model with same production and different cost

- Impossible to distinguish between models based on labor reaction to policy
- No result of ultimatum model hours empirically inconsistent with flexible-hours

Using labor response curve to regulate



flexible-hours: ■ maximizes TS

ultimatum model: ● maximizes TS $\Longrightarrow$ ■ **is local TS minimum**

TS decreasing in minimum wage in at least one model

Remark

If total surplus increasing in minimum wage at w in one model, it's decreasing in other

Wrong model $\implies$ opposite effect of policy on total surplus!

Robust regulation

Why are many real policies nonlinear?

“Best” policy for worker is minimum wage, but **information is limited**

Consider case where regulator

- knows nothing about f, c, but knows *hours* and *compensation*
- knows some specific reduced hours that the worker prefers

Historical motivation

Similar to introduction of overtime pay in the US (1938 Fair Labor Standards Act)

- Regulator knows workers want 40 hour workweek
- No existing regulation

Introducing the regulator

Regulator has no prior over f, c, but

- knows state of market pre-regulation: (ℓ^m, τ^m)
- knows reduced hours, $\hat{\ell} < \ell^m$, preferred by worker at same wage: $(\hat{\ell}, w^m \hat{\ell})$

Worker gets this known preferred contract or better

Regulator's objective: TS maximizing satisficing contract

Offer at least as much utility to worker as known preferred contract

Satisficing

Let $\mathcal{L}[\phi]$ denote the firm's labor choice under regulation ϕ. Policy ϕ is satisficing if for all f, c such that $f'(\ell^m) = c'(\ell^m)$ and $c(\ell^m) = \tau^m$,

$$\max\{\phi(\mathcal{L}[\phi]) - c(\mathcal{L}[\phi]), 0\} \geq w^m \hat{\ell} - c(\hat{\ell})$$

Regulator's objective: TS maximizing satisficing contract

Take satisficing contract that maximizes total surplus in every possible state

TS maximizing

Policy ϕ is TS maximizing if for all f, c such that $f'(\ell^m) = c'(\ell^m)$ and $c(\ell^m) = \tau^m$ and all satisficing ψ,

$$f(\mathcal{L}[\phi]) - c(\mathcal{L}[\phi]) \geq f(\mathcal{L}[\psi]) - c(\mathcal{L}[\psi])$$

This is the least restrictive one

Representation of satisficing policies

Theorem

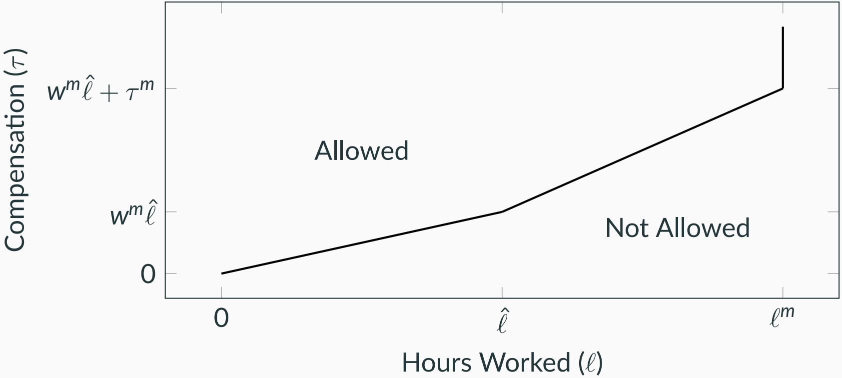
A policy, ϕ, is satisficing if and only if $\phi(\hat{\ell}) = w^m \hat{\ell}$ and

$$\phi(x) \geq \phi_*(x) \equiv \begin{cases} w^m x & \text{if } x \leq \hat{\ell} \\ w^m \hat{\ell} + w^m \frac{\ell^m}{\ell^m - \hat{\ell}}(x - \hat{\ell}) & \text{if } \hat{\ell} < x \leq \ell^m \\ \infty & \text{if } x > \ell^m \end{cases}$$

Least restrictive satisficing regulation, ϕ_*, is TS maximizing:

- Overtime pay with wage multiplier of $\frac{\ell^m}{\ell^m - \hat{\ell}}$ and hours cap at ℓ^m

TS maximizing satisficing policy



- Left of $\hat{\ell}$ is never chosen by firm
- Right of $\hat{\ell}$ is upper bound on cost of additional hours: $c(x) - c(\hat{\ell})$

Intuition behind bound on costs

- Function maximizes disutility of additional hours: $c(x) - c(\hat{\ell})$
- Bound comes from convexity of c and IR of ●

Math BotE

Extensions and future work

- More general bargaining
- Heterogeneous workers
- Competition among firms
- Future work
- **Other paper**

Thank You!

Extensions

More general bargaining

Back

Results

More Example

More general bargaining including Nash and proportional bargaining:

- Minimum wage without loss of optimality
- Efficient, redistributive regulation exists iff overwork in absence of regulation
- Maximizing hours locally minimizes TS iff overwork in absence of regulation

Heterogeneous workers

Softer objective needed for heterogeneous workers

Back

Consider a model where

- Multiple workers have different cost functions, c_i
- Firm contracts with workers individually
- Regulator must apply same ϕ to all workers

Efficiency is too strict with heterogeneous workers!

Need more weight on worker utility

Placing more weight on workers

Back

Regulator maximizes weighted sum of surpluses

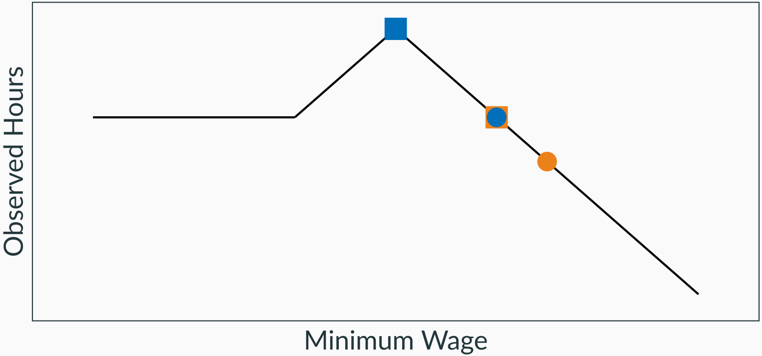
Regulator objective:

Maximize $\alpha u(\ell, w\ell) + (1-\alpha)\pi(\ell, w\ell)$ for $\alpha \in (0.5, 1]$ using ϕ.

Until now, we focused on $\alpha \to 0.5$

Worker surplus maximized by larger minimum wages

Back



flexible-hours: ■ maximizes TS, ■ maximizes WS (can be above or below ●)

ultimatum model: ● maximizes TS, ● maximizes WS

Heterogeneous workers and aggregation

Back

Flexible-hours model convenient for aggregation

- Each hour treated like individual worker
- Hours are fungible across workers

Sometimes convenient to aggregate in ultimatum model too!

Complete information: heterogeneous workers

Back

Ultimatum model result

If regulator maximizes worker surplus of heterogeneous workers

- Optimal regulation is minimum wage
- Representative worker exists
- Optimal policy for representative worker is overall optimal policy
- Representative worker has average costs of all workers affected by policy

Complete information: representative worker intuition

Back

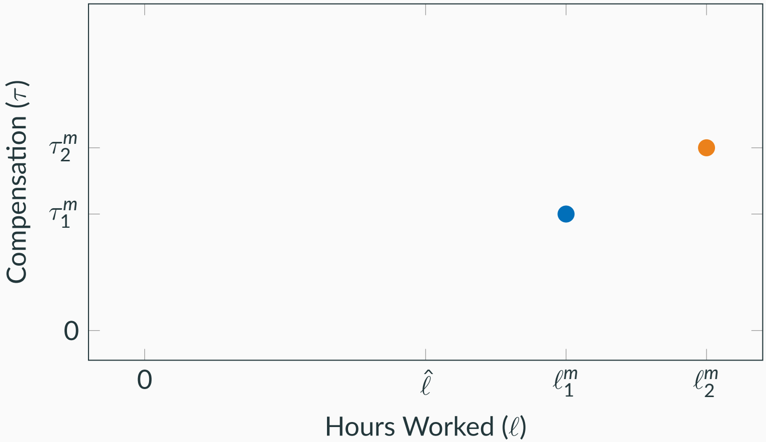
Firm's problem: $\max_{\ell,\tau} f(\ell) - \tau$ s.t. $\tau \geq \phi(\ell)$ and $\boxed{\tau \geq c_i(\ell)}$

Regulation benefits worker $\implies \tau > c_i(\ell) \implies$ contract does not depend on i

Every worker affected by regulation receives same contract!

Robust setting: heterogeneous workers

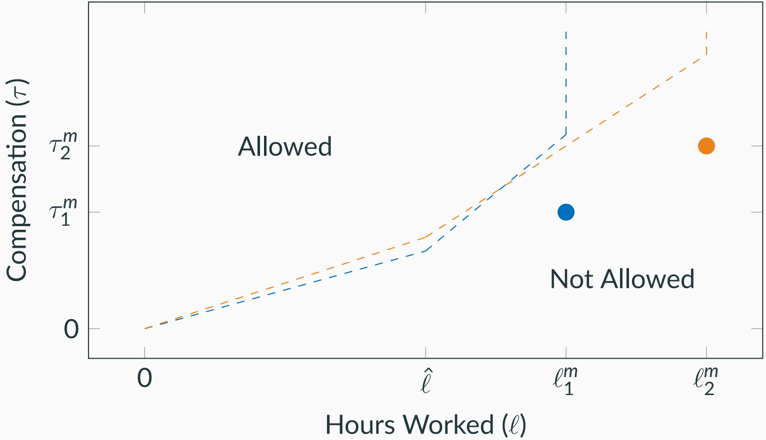
Back



Want TS maximizing satisficing contract for both Worker 1 and Worker 2

Robust setting: heterogeneous workers

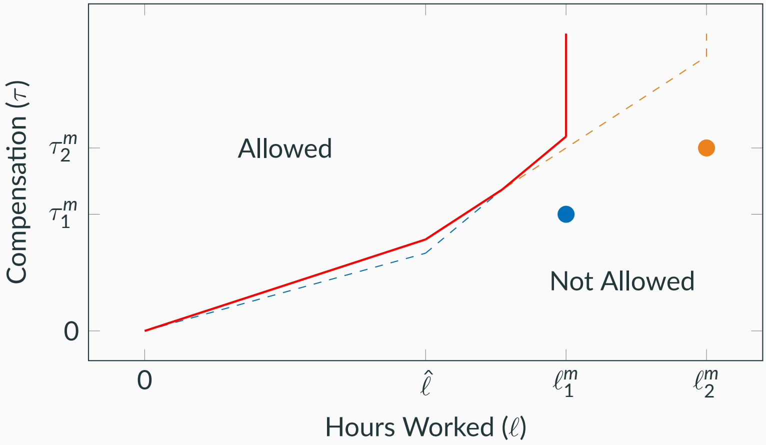
Back



Do procedure for each worker and take maximum

Robust setting: heterogeneous workers

Back



Policy may have multiple levels of overtime – e.g., California and Mexico

Competition among firms

Asymmetric Bertrand competition with potential entrant

Back

Two firms: one incumbent and one potential entrant

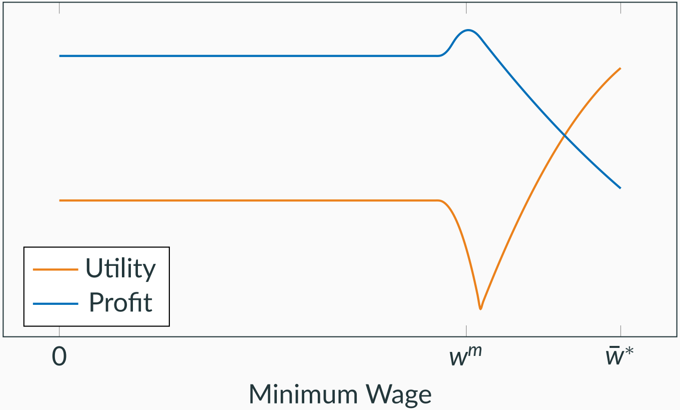
- Entrant has lower marginal productivity than incumbent
- Incumbent moves first with contract offer
- Entrant hires worker if possible to do so profitably

In equilibrium,

- Entrant offers full surplus to worker
- Incumbent matches offer of entrant's maximum surplus

Asymmetric Bertrand competition with potential entrant

Back



Minimum wage weakens competitive pressure by regulating entrant

Asymmetric Bertrand competition with potential entrant

Back

If entrant's wage is lower, minimum wage can reduce incumbent's wage

Asymmetric Bertrand competition: policy implications

Back

Less regulation for new/small firms

- Regulate incumbent without affecting potential entrant
- Not common for pay regulation
- Common for compliance regulations:
  - Americans with Disabilities Act: 15+ employees
  - ACA Shared Responsibility Payment: 50+ employees
  - Equal Employment Opportunity reporting: 100+ employees

Future work

Back

Information design by firms who resist

- hiring more than 40 hours
- paying more than minimum wage

Improving labor caps and overtime policies

- replace hours caps with something softer?

Bargaining design to achieve efficiency

- if contracts are not efficient, how best to improve TS through re-bargaining?

Thank You!

References

Altonji, Joseph G. and Christina H. Paxson (Apr. 1988). "Labor Supply Preferences, Hours Constraints, and Hours-Wage Trade-offs". In: *Journal of Labor Economics* 6, pp. 254–276.

Berger, David W., Kyle F. Herkenhoff, and Simon Mongey (Jan. 2022). *Minimum Wages, Efficiency and Welfare*. Working Paper.

Crépon, Bruno and Francis Kramarz (2002). "Employed 40 hours or not employed 39: Lessons from the 1982 mandatory reduction of the workweek". In: *Journal of Political Economy* 110, pp. 1355–1389.

Feather, Pm and Wd Shaw (2000). "The demand for leisure time in the presence of constrained work hours". en. In: *Economic Inquiry* 38, pp. 651–661.

Flinn, Christopher J. (2006). "Minimum Wage Effects on Labor Market Outcomes under Search, Matching, and Endogenous Contact Rates". en. In: *Econometrica* 74, pp. 1013–1062.

Hamermesh, Daniel S. and Stephen J. Trejo (Feb. 2000). "The Demand for Hours of Labor: Direct Evidence from California". In: *The Review of Economics and Statistics* 82, pp. 38–47.

- Jardim, Ekaterina et al. (2022). "Minimum-wage increases and low-wage employment: Evidence from Seattle". In: *American Economic Journal: Economic Policy* 14, pp. 263–314.
- Manning, Alan (2013). *Monopsony in motion*. Princeton University Press.
- Pandit, Hitanshu (2023). "City Limits: Exploring the relationship between employment and minimum wages using mobile-device location data". In.
- Peters, Hans and Peter Wakker (1991). "Independence of Irrelevant Alternatives and Revealed Group Preferences". In: *Econometrica* 59, pp. 1787–1801.
- Stigler, George J. (June 1946). "The economics of minimum wage legislation". In: *The American Economic Review* 36, pp. 358–365.
- Trejo, Stephen J. (1991). "The Effects of Overtime Pay Regulation on Worker Compensation". In: *The American Economic Review* 81, pp. 719–740.

Appendix

Efficient bargaining framework

Back

Bargaining according to

$$(\ell^*, \tau^*) \equiv \arg\max_{\ell,\tau} M\left(f(\ell) - \tau, \tau - c(\ell)\right) \text{ s.t. } \tau \geq \phi(\ell)$$

$M : \mathbb{R}^2_+ \to \mathbb{R}$ continuous, weakly monotone, and strictly quasiconcave

Alternatively, representation from PO, IIA, and continuity[4] (Peters and Wakker, 1991)

[4] Choice function $C : \Sigma \to \mathbb{R}^2_+$ is continuous if for every sequence, $S_k \to S \implies C(S_k) \to C(S)$

Efficient bargaining example: egalitarian bargaining

Back

Consider egalitarian bargaining

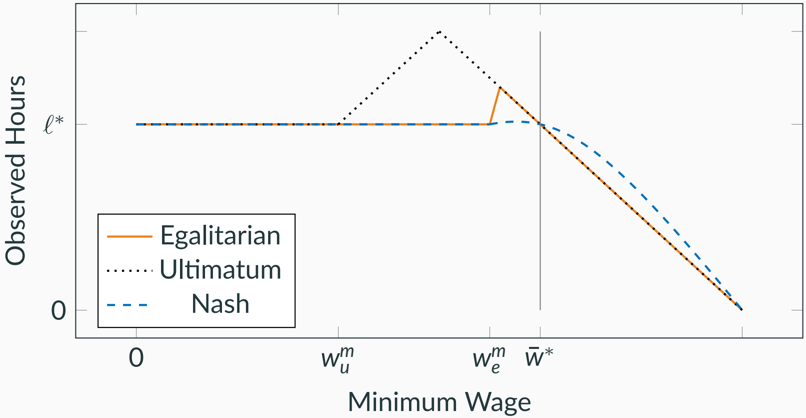
- Assume $-c$ "more concave" than f in that:

$$f(\ell^*) - f'(\ell^*)\ell^* < c'(\ell^*)\ell^* - c(\ell^*)$$

- This implies (and is necessary for) overwork
- The market is described by

$$\max_{\ell,\tau} \min\{f(\ell) - \tau, \tau - c(\ell)\} \text{ s.t. } \tau \geq \phi(\ell)$$

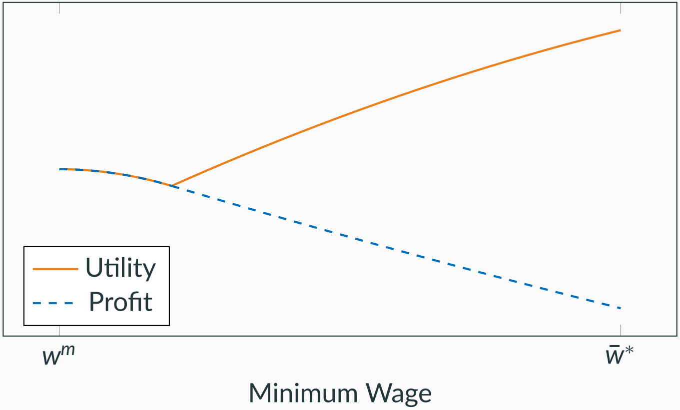
Egalitarian bargaining labor response



Other bargaining frameworks produce similar labor response

Egalitarian bargaining payoffs

Back



Small minimum wages reduce both utility and profit

Derivation of least satisficing regulation

Back

By convexity, for all $x \in (\hat{\ell}, \ell^m)$

$$c(x) - c(\hat{\ell}) < \frac{x - \hat{\ell}}{\ell^m - \hat{\ell}} \left[c(\ell^m) - c(\hat{\ell}) \right]$$

The worker accepted $(\ell^m, \tau^m) \implies \tau^m \geq c(\ell^m)$

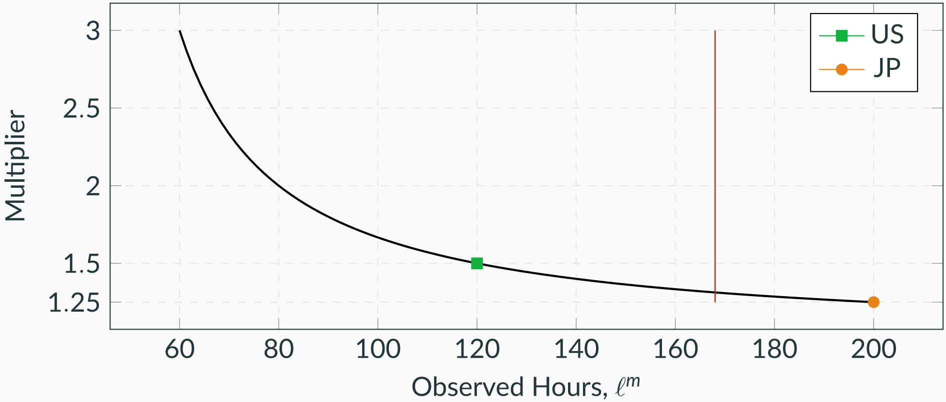
$$\frac{x - \hat{\ell}}{\ell^m - \hat{\ell}} \left[c(\ell^m) - c(\hat{\ell}) \right] \leq \frac{x - \hat{\ell}}{\ell^m - \hat{\ell}} \tau^m$$

Which we rearrange to yield

$$\frac{x - \hat{\ell}}{\ell^m - \hat{\ell}} \tau^m = w^m \frac{\ell^m}{\ell^m - \hat{\ell}} (x - \hat{\ell})$$

Existing policies are below least satisficing

Back



Satisficing policy with kink at 40 hours is above this curve
(there are 168 hours in a week)

BotE Calculation: Overtime in Japan

Suppose that the overtime policy in Japan, which grants time and a quarter after 40 hours of work each week and a cap after 55 hours, is relative maxmin. In this case, $\hat{\ell} = 40$, $\bar{\ell} = 55 \leq \Psi(w^m)$ and

$$1.25 \geq \frac{\Psi(w^m)}{\Psi(w^m) - \hat{\ell}}$$

because the slope of this policy must be at least as large as the LRRM. Last inequality implies

$$\Psi(w^m) \geq 200.$$

We can reject that this policy is satisficing because there are only 168 hours in a week. Therefore, there are possible types of workers that prefer a strict 40 hour cap to this policy.

BotE Calculation: Overtime in the US

Suppose that the overtime policy in the US, which grants time and a half after 40 hours of work, is relative maxmin (ignoring the lack of labor cap). In this case, $\hat{\ell} = 40$ and

$$\frac{\Psi(w^m)}{\Psi(w^m) - \hat{\ell}} \leq 1.5$$

which implies

$$\Psi(w^m) \geq 120.$$

The lack of an hour cap at such a number of hours is irrelevant. This leaves a little under 7 hours for sleep each day. Some workers do work 120 hours on occasion. It is, however, extremely rare.

Asymmetric APA with Spillovers

Motivation: Applications of All-Pay Auctions

Back

Full information all-pay auctions (APA) are used to model:

Lobbying with campaign contributions for political favor

Promotions awarded to employee with greatest output

Patents granted to winner of R&D race

In all cases effort/cost of losers is non-refundable

Motivation: Applications Involve Spillovers

Back

In applications, valuation may depend on bids of other players:

Lobbying campaign contributions affect candidate's election

Promotions output benefits the firm $\implies$ promotion (e.g., to partner) more valuable

Patents (1) loser's research benefits winner and (2) close races require litigation[5]

[5] E.g., Elisha Gray and Alexander Bell fought over invention of the telephone

Motivation: Spillovers enable new applications

Back

Spillovers open up new applications of APA:

Litigation legal expenses may be paid from contested assets

Warfare invasion efforts reduce value of invaded territory

APA with Spillovers: Model

Model (1): Setup

Back

- **Two player** APA with one prize, $v_i : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$, with complete information
- Each player submits score $s_i \in \mathbb{R}_+$ at cost $c_i(s_i)$
- Player i receives payoff:

$$u_i(s_i; s_{-i}) = \mathbf{1}\{s_i \geq s_{-i}\} v_i(s_i; s_{-i}) - c_i(s_i)$$

- Solution concept is Nash equilibrium

Note that we do not care about values of $v_i(s_i; s_{-i})$ for which $s_{-i} > s_i$

Model (2): Timing

Back

Timing of game is:

1. All parameters (v_1, v_2, c_1, c_2) are common knowledge
2. Players submit scores (s_1, s_2) simultaneously
3. Player with the greater score wins
4. Winner (i) receives $v_i(s_i; s_{-i}) - c_i(s_i)$ and opponent ($-i$) gets $-c_{-i}(s_{-i})$

Model (3): Assumptions

Back

Recall that the payoff is: $u_i(s_i; s_{-i}) = \mathbf{1}\{s_i \geq s_{-i}\}v_i(s_i; s_{-i}) - c_i(s_i)$

A1) **Smoothness** $v_i(s_i; y), c_i(s_i)$ continuously differentiable in s_i and $v_i(s_i; y)$ continuous in y

A2) **Monotonicity** for every $s \geq 0$, $c_i'(s_i) > 0$ and $v_i'(s_i; y) < c_i'(s_i)$ for almost all y

A3) **Interiority**

$$v_i(0, 0) > c_i(0) = 0 \quad \text{and} \quad \lim_{s_i \to \infty} \sup_y v_i(s_i; y) < \lim_{s_i \to \infty} c_i(s_i)$$

A4) **Discontinuity at ties**

$$\sup_y v_i(s; y) > 0 \implies v_i(s; s) > 0$$

APA with Spillovers: Results

All-pay auctions without spillovers (quick review)

Back

Find *thresholds*, T_i, largest scores that players can make without negative payoff

$$\mathring{v}_i(T_i) = c_i(T_i)$$

All-pay auctions without spillovers (quick review)

Back

Find *thresholds*, T_i, largest scores that players can make without negative payoff

$$\mathring{v}_i(T_i) = c_i(T_i)$$

Only player with largest threshold has positive payoff

$$u_i = \max\{0, \mathring{v}_i(T_{-i}) - c_i(T_{-i})\}$$

All-pay auctions without spillovers (quick review)

Back

Find *thresholds*, T_i, largest scores that players can make without negative payoff

$$\mathring{v}_i(T_i) = c_i(T_i)$$

Only player with largest threshold has positive payoff

$$u_i = \max\{0, \mathring{v}_i(T_{-i}) - c_i(T_{-i})\}$$

Look for mixed strategy equilibrium with support on $[0, \min\{T_1, T_2\}]$

$$G_{-i}(s_i)\mathring{v}_i(s_i) - c_i(s_i) = u_i$$

All-pay auctions without spillovers (quick review)

Back

Only player with largest threshold has positive payoff

$$u_i = \max\{0, \mathring{v}_i(T_{-i}) - c_i(T_{-i})\}$$

Look for mixed strategy equilibrium with support on $[0, \min\{T_1, T_2\}]$

$$G_{-i}(s_i)\mathring{v}_i(s_i) - c_i(s_i) = u_i$$

Solve indifference condition for G_{-i}

$$G_{-i}(s_i) = \frac{u_i + c_i(s_i)}{\mathring{v}_i(s_i)}$$

Difficulty of incorporating spillovers

Back

1. Threshold depends on equilibrium distribution

$$\int_0^{T_i} v_i(T_i; y)\, dG_{-i}(y) = c_i(T_i)$$

2. Not easy to solve indifference condition for G_{-i}

$$\int_0^{s_i} v_i(s_i; y)\, dG_{-i}(y) - c_i(s_i) = u_i$$

Difficulty of incorporating spillovers

Back

1. Threshold depends on equilibrium distribution

$$\int_0^{T_i} v_i(T_i; y)\, dG_{-i}(y) = c_i(T_i)$$

Solution: ignore thresholds and start with indifference condition

2. Not easy to solve indifference condition for G_{-i}

$$\int_0^{s_i} v_i(s_i; y)\, dG_{-i}(y) - c_i(s_i) = u_i$$

Solution: use Volterra integral equations

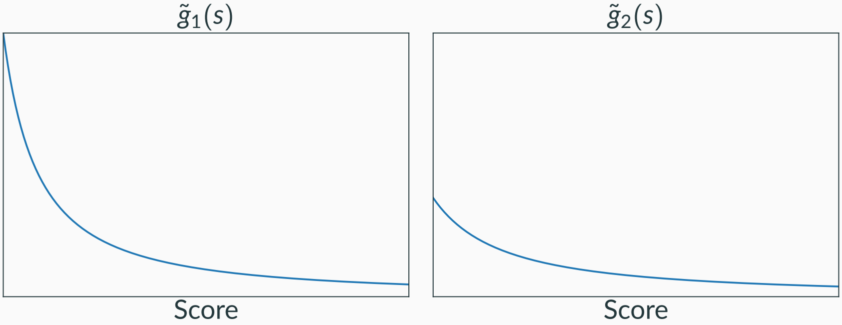
Outline of construction (Step 1)

Back

There exist *unique* continuous functions $(\tilde{g}_1, \tilde{g}_2)$ that solve

More

$$\int_0^s v_{-i}(s; y)\tilde{g}_i(y)dy - c_{-i}(s) = 0$$



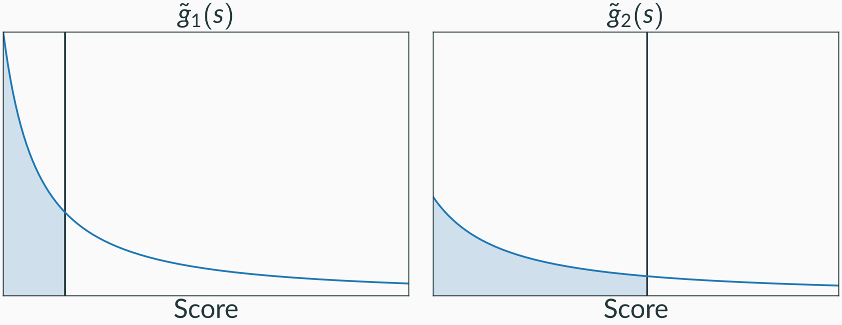
Outline of construction (Step 2)

Back

There exists an $\bar{s}_i$ such that $\tilde{g}_i(x)$ is *positive* for $x \leq \bar{s}$ and

More

$$\int_0^{\bar{s}_i} \tilde{g}_i(y)dy = 1$$



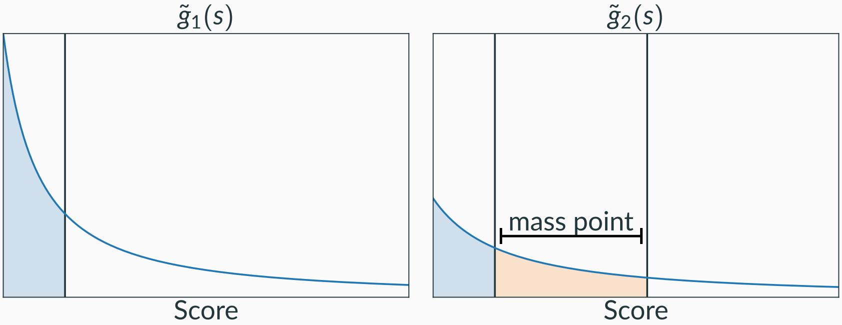
Outline of construction (Step 3)

Back

Equilibrium unique with support up to $\bar{s} = \min_i \bar{s}_i$ and distributions

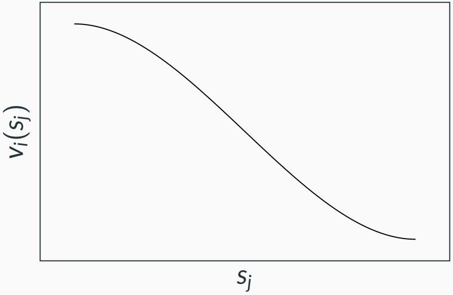
More

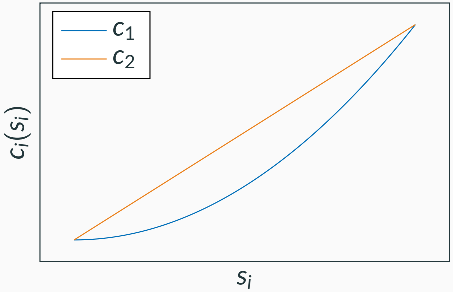
$$G_i(x) = \int_0^x \tilde{g}_i(y)dy + \int_{\bar{s}}^{\bar{s}_i} \tilde{g}_i(y)dy$$



Reversal result: ranked costs not sufficient

Back





Ranked costs **not** sufficient to determine dominant player with spillovers

Example

Player 1 has absolute advantage, but Player 2 has comparative advantage over high bids

Player 2 places more density on high bids $\implies$ prize less appealing to Player 1

Thank You!

APA with Spillovers: Appendix

Invertibility (Step 1)

Back

Want to invert linear operator

$$T[f](s) = \int_0^s v(s;y)f(y)dy$$

Volterra theorem guarantees invertibility and gives sequential solution

$$\tilde{g}_{n+1}(s) = \frac{1}{v(s;s)}\left(c'(s) - \int_0^s v'(s;y)\tilde{g}_n(y)dy\right)$$

with initial condition $\tilde{g}_0(s) \equiv 0$

Invertibility: Intuition

Back

Consider our original equation

$$\int_0^s v(s;y)\tilde{g}(y)dy = c(s)$$

and consider this 3×3 discrete approximation of this problem for $s \in [0,1]$

$$\frac{1}{3}\underbrace{\begin{bmatrix} v(1/3, 1/3) & 0 & 0 \\ v(2/3, 1/3) & v(2/3, 2/3) & 0 \\ v(1, 1/3) & v(1, 2/3) & v(1,1) \end{bmatrix}}_{\mathbf{V}} \cdot \underbrace{\begin{bmatrix} \tilde{g}(1/3) \\ \tilde{g}(2/3) \\ \tilde{g}(1) \end{bmatrix}}_{\mathbf{g}} \approx \underbrace{\begin{bmatrix} c(1/3) \\ c(2/3) \\ c(1) \end{bmatrix}}_{\mathbf{c}}$$

The matrix $\mathbf{V}$ is triangular and A4 guarantees that the diagonal is positive

Therefore, the matrix is invertible and $\mathbf{g} = 3\mathbf{V}^{-1}\mathbf{c}$

The solution is a PDF (Step 2)

Back

Convergent finite definite integral cannot diverge: $g_i(0) = \frac{c_k'(0)}{v_{-i}(0;0)}$

Positive $\tilde{g}_i(s) > 0$ on the relevant interval, $\{s : \int_0^s |\tilde{g}_i(y)|dy \leq 1\}$

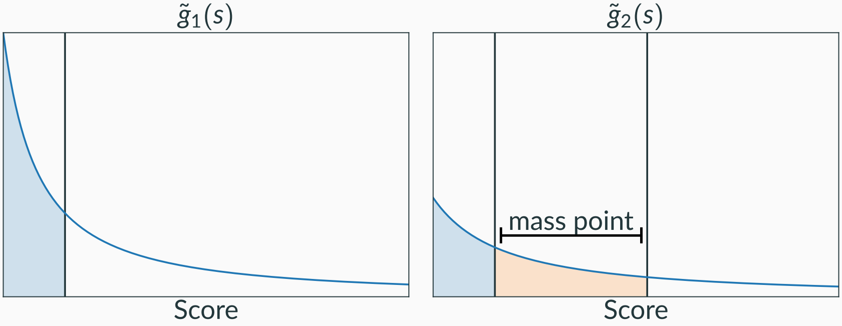
Cutoff exists cannot integrate to a number less than one because

$$c_{-i}(s) = \int_0^s v_{-i}(s;y) g_i(y) dy \leq \left(\int_0^s |g_i(y)| dy \right) \left(\max_{y \in [0,s]} v_{-i}(s;y) \right)$$

so $\int_0^s |g_i(y)|dy \geq \frac{c_{-i}(s)}{\max v_{-i}(s;y)}$ greater than 1 for $s \to \infty$ (A3)

Uniqueness (Step 3)

Back



- Volterra theorem ensures uniqueness of smooth part
- This is unique mass-point that yields common support

Ranked costs counterexample

Back

For example, consider

$$v(s;y) = \frac{11}{10} + \frac{y^3}{3} - \frac{y^2}{2}$$

with $c_1(s) = s^2$ and $c_2(s) = s$.

$$\tilde{g}_1(s) = \frac{1}{\frac{11}{10} + \frac{s^3}{3} - \frac{s^2}{2}} \qquad \tilde{g}_2(s) = \frac{2s}{\frac{11}{10} + \frac{s^3}{3} - \frac{s^2}{2}}$$

Integrating these shows that $\int_0^1 \tilde{g}_1(s)ds < 1$ and $\int_0^1 \tilde{g}_2(s)ds > 1$ so $\bar{s} < 1$, player 1 has an mass-point, and player 2 receives a positive payoff.